Randomized Competitive Algorithms for Admission Control in General Networks*

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Admission control problems arise in very fast networks whenever there is a request to send a large amount of data from one node in the network to another node. The challenge is to decide (on-line) whether or not the network can or should accomodate the request. In this work we provide *randomized* algorithms for *general networks* and for the case where each request asks for the entire bandwidth of the virtual circuit(s) that are possible to be granted. Let L be the length of the longest simple path in the network, and $k(G)$, $\lambda(G)$ the vertex and edge connectivity of G.

Admission control has been previously considered in a variety of contexts (see e.g. [ABFR, 94], [AGLR, 94]). The problem was formulated in [GG, 92]. Competitive algorithms for general networks were provided in [AAP, 93], but with the restriction that every virtual circuit requests at most $1/\log n$ of the capacity of the lowest capacity link. If all virtual circuits have infinite duration, then the competitive ratio of the algorithm in [AAP, 93] is $O(\log n)$. As [AGLR, 94] note, the analysis of [AAP, 93] can be easily modified to show a competitive ratio of $O(\log L)$, for connections that use at most $1/\log 2L$ of the available bandwidth, where L is the longest simple path in the network. Recently, A. Fiat, S. Leonardi and Y. Bartal had an unpublished lower bound of n^{ϵ} for deterministic algorithms. This bound holds for a special "brick wall" network of maximum degree 3.

We model the underlying network by a graph $G(V,E)$ whose nodes represent end-stations or switches and whose links represent network connections of high bandwidth. Let $n = |V|$. A *request* r is a pair $\{u, v\}$ of nodes which must be connected by a *virtual circuit* i.e. an available simple path in G connecting u, v. We focus on the situation where each request asks for the *entire* link bandwidth and on the case where established virtual circuits have *infinite* duration. We concentrate on *oblivious* cases.

Our protocol, in response to a request, may declare some edges to be *forbidden.* Forbidden edges block future requests whose selected paths use that edge. A particular simple path between u, v is called *free* if *no edge* (or vertex) of the path is forbidden or used by an already accepted request.

The protocol then is as follows: Let $r = \{u, v\}$ be a particular request. If no simple path between u, v is free, then reject the request. Else, r is a *candidate.* If a request becomes a candidate then accept it with fixed probability p (e.g. 1/2), and select at random one of the free simple paths connecting u, v. Else reject the request and choose all of the edge disjoint free paths between u, v. Let $L(\pi)$ the length of such a free path (π). Pick a random integer l uniformly in $[1, \log L(\pi) + 1]$. Consider π. Number its edges $1, 2, \ldots$ along π from u to v. Declare *forbidden* the edges on π of numbers $i2^l$ for all $0 < i \leq L(\pi)/2^{l-1}$. The *forbidden* edges partition the selected path of $\{u, v\}$ into equal length segments. Also, if any vertex on the selected path is a cut-point, we declare forbidden some edges not in the path, out of that vertex (extra forbidden edges).

We develop a partition of C_i into *classes.* In particular, some optimal request f will be in a class of a rejected candidate c if its chosen path overlaps significantly some path of c. Here, *significant overlap* means that f's selected path overlaps some c's path in a free segment s and to an extend that it would be split if an edge exactly in the middle would be declared forbidden. We say f is in the *class*(s, c).

We prove the following result:

For any G of degree at least $\log n$ our protocol achieves an expected competitive ratio of $O(\log L \cdot \frac{k(G)}{\lambda(G)})$, which is $o(\log L)$ for many graphs.

A basic element of the proof is the calculation of the probability that an optimal request goes to a particular class.

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References

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