# Power Consumption Analysis in DUDe 5G MIMO Networks

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Abstract— The advent of 5G technology has brought unprecedented advancements in wireless communication, promising higher data rates and improved user experiences. However, the implementation of Multiple-Input Multiple-Output (MIMO) techniques in 5G networks has significantly increased power consumption, posing a challenge to sustainability and operational costs. This study conducts a comparative analysis of energy efficiency in 5G MIMO networks, specifically between Downlink Uplink Coupling (DUCo) and Downlink Uplink Decoupling (DUDe) techniques. By examining the remaining power in the Base Stations (BSs) of the implemented Heterogeneous Network (HetNet), this analysis evaluates the potential of DUDe over DUCo in managing power consumption. The findings of this study aim to assist network operators, researchers, and policymakers in developing greener and more sustainable 5G infrastructures for a cleaner, energyefficient future.

Keywords — Multiple Input and Multiple Output (MIMO), Power Consumption, Energy Efficiency, Heterogeneous Networks, Downlink Uplink Decoupling (DUDe).

## I. INTRODUCTION

Wireless communication has undergone swift and significant changes, in the latest years, leading to the introduction of 5G Heterogeneous Networks (HetNets). This advancement leads to a new age of exceptional connectivity and opportunities fueled by data. Cutting-edge technologies such as Multiple-Input Multiple-Output (MIMO) systems are at the heart of 5G's revolutionary features.

MIMO has become a critical technology in contemporary wireless networks, utilizing multiple antennas at both sending and receiving ends. This approach allows for the concurrent transmission of several data streams, exploiting spatial diversity and multipath propagation to overcome the challenges of fading and interference, thereby improving spectral efficiency, increasing the capacity of networks, and facilitating uninterrupted communication across a growing network of connected devices.

In 5G networks, MIMO technology has undergone substantial advancements, encompassing various configurations such as 2x2, 4x4, 8x8, and Massive MIMO with hundreds of antennas. These configurations enable network operators to serve diverse user demands, from small-scale indoor environments to large-scale outdoor

deployments, and support the ever-growing number of connected devices [1], [2], [3].

However, amongst the numerous benefits of 5G HetNets lies a concerning obstacle, the increase in power consumption accompanying the implementation of MIMO in 5G networks. The use of multiple antennas, coupled with complex signal processing operations, inherently demands increased energy resources, presenting a critical challenge to the overall sustainability of 5G networks. With the exponential increase in connected devices and data traffic, the power efficiency of 5G MIMO networks has become a focal point for network operators, service providers, and researchers alike.

Effectively managing power consumption is not only essential for the economic sustainability of telecommunication companies but also for environmental sustainability. As the world becomes more aware of the need for energy efficiency and reducing carbon emissions, optimizing energy use in telecommunications not only supports economic goals but also aligns with global environmental priorities [4], [5], [6], [7].

Thus, understanding and addressing the intricacies of power consumption in 5G MIMO networks has become mandatory to ensure efficient network operation and the reduction of ecological impact. By examining the underlying factors that contribute to energy inefficiencies and exploring various strategies, power-saving mechanisms, and energy-efficient algorithms, this research stands out because it closely examines how the Downlink Uplink Decoupling (DUDe) technology can improve 5G mixed MIMO networks, compared to the traditional Downlink Uplink Coupling (DUCo). Our previous research [8] provides a more detailed explanation of the distinctions between these two technologies, similarly as research [9].

It offers a new way of thinking about how to allocate resources by treating uplink and downlink channels separately. The main aim is to use resources better in 5G MIMO networks, leading to higher efficiency and smoother, interference-free communication for users. It especially focuses on cutting down how much power each base station uses.

Reviewing the existing literature reveals several notable endeavors with similar aims of enhancing power consumption in HetNets MIMO 5G networks. Firstly,

optimizing 5G networks to meet the increasing demands for high data rates and low-latency communication is a critical challenge in telecommunications. Addressing this, a study offers insights into effective strategies for 5G optimization, using Deep MIMO platform, focusing on enhancing network performance and efficiency. This research is a key resource for understanding the potential of 5G technologies and their implementation in real-world scenarios [10].

Furthermore, the transformative potential of beamforming techniques in 5G networks, particularly in enhancing signal precision and network capacity, is thoroughly examined in a recent study [11]. By detailing the application and benefits of these strategies, the research underscores their critical role in the next generation of telecommunications networks, offering valuable insights for the advancement of 5G technology.

The rest of the paper is organized as follows: in Section II, the introduction of the mathematical model utilized in the simulation setup is presented. Furthermore, in Section III, a detailed analysis is conducted on the algorithms that form the foundation for constructing the experimental scenarios. Section IV proceeds to delineate the specifics of the simulation setup and the methodologies employed to assess the performance of DUDe in the MIMO 5G HetNets. Subsequently, Section V presents the results of the simulations and conducts a comprehensive analysis of the findings. Finally, Section VI concludes the paper by synthesizing conclusions drawn from the findings and highlighting potential avenues for future research.

#### II. MATHEMATICAL MODEL

In this part, a detailed explanation of the mathematical framework used in the experiments is provided. Initially the shortest distance between User Equipments (UEs) and different Base Station (BS) antennas is determined by using the model described in Section 7.4.1 of TR 38.901 [12]. It's important to mention that a deep dive into the factors mentioned next is beyond the scope of this paper.

The Path Loss (PL) model, particularly for Urban Macro (UMa) - Line of Sight (LOS) scenarios, can be expressed as follows:

$$PL_{\text{RMa-LOS}} = \begin{cases} PL_1 & 10m \le d_{2D} \le d_{\text{BP}} \\ PL_2 & d_{\text{BP}} \le d_{2D} \le 10 \text{km} \end{cases}$$
(1)

$$\begin{array}{l} PL_1 \\ = 20 \log_{10}(40\pi d_{3\mathrm{D}}f_c/3) \\ + \min(0.03h^{1.72}, 10) \log_{10}(d_{3\mathrm{D}}) \\ - \min(0.044h^{1.72}, 14.77) \\ + 0.002 \log_{10}(h)d_{3\mathrm{D}} \end{array} \tag{2}$$

$$PL_2 = PL_1(d_{BP}) + 40 \log_{10}(d_{3D}/d_{BP})$$
 (3)

To find the best antenna for connecting, the Signal-to-Noise Ratio (SNR) was utilized. This basically compares how strong the signal is to how much background noise there is. In the experiments, both the signal and the noise were calculated at the same places in the system and within the same range of frequencies. Then, the following math formula is used, to figure out the SNR:

$$SNR = Psignal/Pnoise$$
 (4)

For the comparative analysis of power consumption between DUDe and the DUCo approaches, a critical aspect of the mathematical model involves converting SNR values from a logarithmic decibel scale to a linear milliwatt scale. This conversion is fundamental to understanding the real-world implications of the SNR on power consumption. Pnoise symbolizes the total Noise of environment.

The formula used for this transformation is given by:

$$P(W) = 10^{(SNRdb - 30)/10}$$
 (5)

SNRdb is the SNR value in decibels. This formula is derived from the definition of a decibel, which is a dimensionless unit that expresses the ratio of power on a logarithmic scale. By converting SNR to a linear scale, the power required for a given SNR threshold can be directly computed, facilitating a precise comparison between the power efficiencies of DUDe and DUCo technologies.

In the experiments, this conversion model allowed the accurate determination of the power consumption for both downlink and uplink channels across multiple antenna indexes. By applying this formula, the gap between the abstract, theoretical SNR values and their tangible, practical power consumption equivalents were bridged. Consequently, the power efficacy of both DUDe and DUCo were able to be assessed, under varying network conditions, providing a robust framework for the analysis. This methodology underscores the necessity of incorporating real-world units and measurements into theoretical models to yield actionable insights and recommendations for system design and deployment in telecommunications.

## III. ALGORITHM ANALYSIS

This section contains an examination of the algorithm that was used to develop the code for the experiments. Here, a comparison between the experiment's findings and the theoretical algorithm that was devised can be seen, to confirm that the conclusions align.

## Algorithm 1 Algorithm for MIMO Power Consumption

- 1. Initialize the environment.
- 2. Load the 'dataset.mat' file.
- 3. Read parameters from 'parameters.m' file.
- 4. Set K as an array from 1 to 64.

- 5. Calculate the maximum number of active subcarriers.
- 6. Initialize variables for base stations, users, and antenna gains.
- 7. Calculate the total number of users in the dataset.
- 8. Set the number of base station antennas and transmit power.
- 9. Initialize variables for data sums and counters.
- 10. Display the number of users.
- 11. Calculate bandwidth and noise power.
- 12. Calculate SNR.
- 13. Initialize matrices for distance and pathloss between BS and users.
- 14. For each base station, calculate distances and pathloss for each user.
- 15. Sort pathloss values for each base station.
- 16. Initialize variables for deep learning model input.
- 17. Initialize a struct array 'dynamic pathloss bs ue'.
- 18. Populate 'dynamic\_pathloss\_bs\_ue' with values from 'pathloss\_bs\_ue'.
- 19. Initialize capacity for base stations in bps.
- Sort 'dynamic\_pathloss\_bs\_ue' by SNR and base station.
- 21. Initialize matrices and variables for capacity allocation.
- Assign capacity to users based on requirements and SNR.
- Iterate through users to find the best base station for allocation.
- 24. Calculate throughput for successfully allocated users.
- 25. Sort 'success\_throughput' by user numbers.
- 26. Calculate and reshape power SNR values for uplink and downlink.
- 27. Initialize variables for DUDe and Coupling scenarios.
- 28. For each group, allocate capacity and handle overflow cases.
- 29. Adjust power arrays for DUDe and Coupling.
- 30. Count users for DUDe and Coupling per group.
- 31. Plot remaining power per base station for DUDe and Coupling.
- 32. Plot number of UEs per base station for DUDe and Coupling.
- 33. Calculate average data rate per user and total sum of data rates.
- 34. Store average and total throughput in 'success throughput' struct.

The outlined algorithm serves as an abstract representation of an algorithm used to assess the performance of telecommunication networks, particularly focusing on the power consumption and service distribution between DUDe and a more traditional approach DUCo. The algorithm begins with the initialization of the MATLAB environment, setting the stage for the subsequent data loading and variable declarations which are pivotal for the experiments.

The core of the algorithm involves iterating through a dataset of BS and users, where for each pairing, it computes the Euclidean distance and pathloss. This information feeds into the calculation of SNR, which is crucial for

understanding the quality of the communication link between the BS and users. These SNRs are then sorted to prioritize users based on the strength of their downlink and uplink channels.

Furthermore, the algorithm checks if the BS has enough power capacity to fulfill the user's service demands, thereby ensuring that resources are efficiently distributed. The struct 'allocatedStruct' is updated to save the allocations made, containing the required power in mWatt per user.

Moreover, the algorithm details how remaining power is calculated post-allocation, providing a clear comparison between the DUDe and DUCo scenarios in terms of power efficiency. This is visualized through bar graphs that illustrate the remaining power and number of users served per base station, offering a visual representation of the algorithm's outcome.

Finally, with the computation of the average and total throughput achieved in the network, encapsulating the performance metric crucial for network operators and designers. This comprehensive approach laid out in the algorithm provides a structured way to simulate and analyze the performance of different network configurations, providing insights that are instrumental for optimizing power consumption and service distribution in telecommunication systems.

#### IV. SIMULATION ENVIRONMENT

This section offers an in-depth examination of the network configuration and the critical details associated with it. It's important to note that both the network structure and the data used for the experiments were sourced from the DeepMIMO platform's website [13]. This platform serves as a valuable resource, offering the necessary infrastructure to shape and execute the experiments effectively.

More specifically there is a 5G MIMO HetNet setup as seen in Fig. 1. It contains an urban setting where the main street, stretching horizontally, spans 600 meters in length and 40 meters in width. a vertical counterpart spanning 440 meters in length and 40 meters in width. Similar to the main street, buildings line both sides. Along the main street, there is uniformity, as all buildings share bases with dimensions of 30 meters by 60 meters. On the other hand, the second street exhibits a distinct configuration, with buildings standing on bases measuring 60 meters by 60 meters.

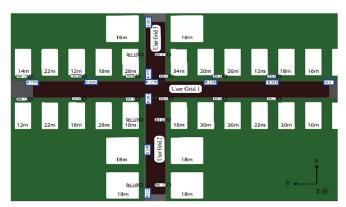


Fig. 1. General topology of simulated network.

Additionally, there is a total of 18 BSs named BS1 through BS18 installed, all standing at a height of 6 meters. Along the main street, 12 of these stations are placed, from BS1 to BS12 with 6 stationed on each side. Regarding the spacing arrangement, there is a 52-meter gap between the BS on one side of the street and those on the opposite side. Breaking it down further, there is a 100-meter separation between clusters, BS1, BS3, and BS5; BS2, BS4, and BS6; BS7, BS9, and BS11; BS8, BS10, and BS12.

Furthermore, on the second street, BSs 13 through 18 are strategically positioned. Each side of the street contains three BS units, spaced 150 meters apart, BS13, BS15, and BS17, and similarly BS14, BS16, and BS18.

A closer examination reveals a consistent gap of 52 meters between neighboring stations, specifically BS13 and BS14, BS15 and BS16, as well as BS17 and BS18. These placements and dimensions ensure the network's seamless integration into the urban fabric, promising robust connectivity across the area.

Additionally, there are three distinct User Grids (UG), UG1, UG2, and UG3 which play a crucial role in being able to serve a substantial total of up to 1,184,923 UEs. The first UE in each grid claims the distinction of having the lowest (x, y) coordinates. Regarding the height, all UE grids maintain a consistent 2-meter elevation.

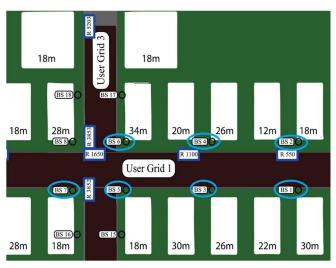


Fig.2. Simulations' topology.

UG1 stretches horizontally along the main street for 550 meters with a width of 35 meters. It begins 15 meters from the street's start and extends nearly to its end. Containing 2751 rows, each row accommodates 181 UEs at equal y-coordinates, separated by 20 cm, resulting in a total of 497,931 UEs. UG2, on the other hand, takes up rows 2752 to 3852, a total of 1101 rows host 181 UEs each, maintaining a 20 cm gap between neighbors totaling 199,281 UEs. Lastly, UG3 consists of rows 3853 to 5203, accommodating 1351 rows with 361 UEs per row. With a 10 cm spacing between UEs, UG3 has 487,711 UEs.

For the experiments, specific areas are chosen to implement the setups. In Fig. 2, the selected locations are presented: User Grid 3 will be served by BS17, while User Grid 1 will be covered by BS4, BS3, BS5, and BS6. The

transmission power of the BSs is set at 45 dBm, with a gain of 21 dBi. To explore different user scenarios, three setups with 181, 362, and 543 users were conducted, all maintaining a constant UE power of 20 dBm. A summary of these network parameters can be found in Table I.

TABLE I. SIMULATION PARAMETERS

Parameter	Value
Transmit power(dbm)	45 dbm
BS height (m)	6 m
BS/UE gain (dbi)	21 dbi, 0 dbi
Power Consumption (MWats)	1000000
NumberOfUEs	181,362,543
Power Noise	Pnoise= -74+10log(Bandwidth(hz))

#### V. SIMULATION RESULTS

The graphs from Fig. 3 to Fig. 8 show how DUDe outperforms DUCo in distributing UEs and managing power consumption across a 5G MIMO network. This analysis compares the two approaches across different UE densities, ranging from 181 to 543 users. It's evident that DUDe consistently offers better allocation and power efficiency compared to DUCo in these scenarios.

From Fig. 3 and Fig. 4, the power consumption per UE for the DUDe across antennas 1 to 7 is approximately [0.45, 0.225, 0.04375, 0.05, 0.11667, 0.05, 0.225] mW, respectively. Average power consumption is approximately 0.08906 mW. The power consumption per UE for DUCo across antennas 1 to 7 is approximately [0.45, 0.225, 0.05625, 0.045, 0.1125, 0.06667, 0.15] mW, respectively. The average power consumption per UE is approximately 0.09118 mW.

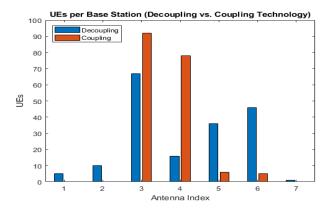


Fig.3. UEs distribution for 181 UEs.

On average, the DUDe technology is more power-efficient per UE than the coupling technology, with the average power consumption per UE being lower for DUDe (0.08906 mW) than for coupling (0.09118 mW). At antenna 6, DUDe is both more power-efficient and supports more UEs than coupling, indicating better performance in the scenario.

For antenna index 4, where coupling supports more UEs (78 compared to 16 for DUDe), DUDe still maintains a lower power consumption per UE, which indicates that DUDe is using power more efficiently, even though it supports fewer UEs in this instance.

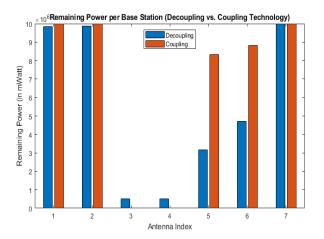


Fig.4. Power consumption for 181 UEs.

Observing Fig. 5 and Fig. 6, which correspond to the 362-user scenario, further validates the enhanced performance of DUDe. Fig. 5 showcases DUDe's capability to support a higher number of UEs, without a proportional increase in power usage as depicted in Fig. 6. The nuanced power management of DUDe becomes evident here, where despite doubling the user load from the previous scenario, the increase in power consumption remains marginal, emphasizing the scalability of DUDe technology.

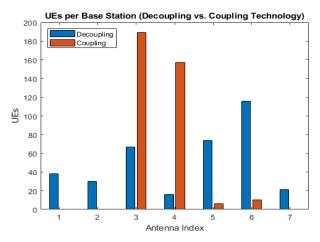


Fig.5. UEs distribution for 362 UEs.

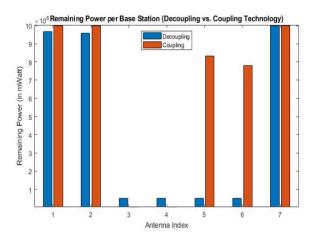


Fig.6. Power consumption for 362 UEs.

Lastly, Fig. 7 and Fig. 8, showcase the 543-user scenario. Fig. 7 reinforces the adaptability of DUDe to manage even higher user densities effectively, particularly at antenna index 4, which emerges as a high-capacity point in the network. This adaptability is mirrored in Fig. 8, where the power consumption of DUDe, while understandably higher due to increased user density, remains consistently efficient. Even at antenna index 6, where DUDe accommodates a larger user load, its power consumption is on par with less populated antennas, showcasing DUDe's efficiency in power management and user distribution across the antennae.

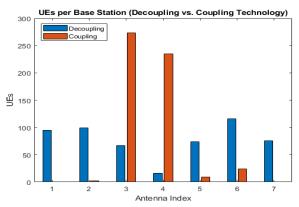


Fig.7. UEs distribution for 543 UEs

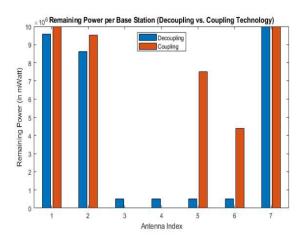


Fig.8. Power consumption for 543 UEs.

In assessing the power effectiveness and UE distribution capabilities of DUDe technology, the experiments centered on determining the remaining power per BS and the UE distribution across various antenna indexes.

The initial dataset portrayed DUDe as being marginally more power-efficient on average compared to the coupling technology. Notably, antennas that supported a higher number of UEs for DUDe also tended to exhibit lower power consumption per UE, signifying an enhanced performance in specific configurations.

The numerical data in these analyses paints a clear picture: DUDe technology not only supports a larger number of users across various network loads but does so with a increased efficiency in power consumption. In every scenario assessed, DUDe has shown it can handle the user allocation in 5G MIMO networks more efficiently than DUCo. This efficiency is crucial as the telecommunications industry progresses toward denser networks and higher throughput demands, positioning DUDe as a compelling solution for future 5G deployments.

Compiling the results from the scenarios examined proves DUDe's capability for power efficiency in 5G MIMO networks. The results reveal that DUDe technology not only excels in managing power but also demonstrates remarkable flexibility across a variety of network loads.

The power efficiency of DUDe is impressively consistent, even as the number of UEs scales up. This consistent efficiency is evident in several antenna configurations, where DUDe technology consistently outperforms traditional coupling approaches.

These findings highlight the importance of leveraging DUDe's strengths in various operational scenarios, with even more network load and UEs. The performance of DUDe, while consistently strong, can be maximized when configured to the network's specific conditions and needs.

The effective application of DUDe in the telecommunications infrastructure showcases its potential to enhance network performance across a variety of scenarios. Precision in data analysis further supports these conclusions, as the utilization of exact numerical data confirms DUDe's superior performance, reducing the uncertainty often associated with visual data interpretation and reinforcing the validity of this scientific approach.

#### VII. CONCLUSIONS AND FUTURE WORK

This study provides a thorough investigation into the energy consumption patterns in the 5G MIMO HetNets, focusing on comparing the energy efficiency between DUDe and DUCo.

This investigation was conducted by a series of methodically designed experiments, all aimed at analyzing the relationship between the operational efficiency of these networks and their associated power demands.

These experiments revealed numerous factors that play a significant role in the power consumption. Notably, the results indicate that DUDe configurations outperform DUCo in terms of power efficiency.

This discovery underscores the potential for significant energy savings and supports a strategic approach for designing and improving networks. By prioritizing energy efficiency without sacrificing service quality or performance, these findings support the advancement toward more sustainable and environmentally friendly 5G network operations.

Looking forward to this, there are many opportunities for further exploration and innovation. Future research can benefit from looking into adaptive beamforming, a technique that could make transmissions more focused towards the UEs, possibly even saving energy.

Furthermore, adding artificial intelligence and machine learning means networks could automatically adjust to be more efficient based on how much data they are handling and other factors. Additionally, technologies like massive MIMO and millimeter-wave (mmWave) communications could make networks faster and more efficient. However, their impact on the network's power consumption profile remains a critical area of investigation.

Also, further research will explore these ideas to create next-gen networks that are not just faster and stronger but also better for the planet. Finally field test and real-world implementations to further strengthen the results of this research.

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