Deploying AL-FEC with Online Algorithms

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***Abstract*—Application Layer Forward Error Correction (AL-FEC) schemes are the most suitable mechanism for error control in mobile multicast services. AL-FEC introduces redundant information in advance with the source data to provide reliability control in the multicast transmission. Due to this fact the introduced redundancy must be carefully selected with respect to the current network conditions to avoid channel bandwidth wastage and achieve an efficient and reliable multicast delivery. However, the efficient selection of the introduced redundancy is not a trivial issue for the multicast source, due to the individual constraints of a multicast environment. In this work, we present a novel way to face the AL-FEC deployment utilizing online algorithms for selecting the appropriate amount of redundancy introduced in the multicast transmission. We provide an efficient way to apply AL-FEC protection on mobile multicast environments with randomized online algorithms eliminating the need for prior knowledge of the network conditions. To this direction, we present a competitive framework under which, we state the problem of the efficient deployment of AL-FEC protection and provide a randomized online algorithm for the AL-FEC application over mobile multicast environments.**

***Keywords*-forward error correction, reliability control, raptorq codes, online algorithms, competitive analysis**

I. Introduction

Forward Error Correction (FEC) is a protection method against packet losses adopted in several multicast standards. FEC concept, unlike the common methods of error control (e.g. ARQ, Carousel), is based in its "forward" feature where redundant information is transmitted in advance with the source data in order to enable a receiver to overcome data erasures, making FEC a feedback-free mechanism. The feedback-free feature of FEC perfectly matches the individual constraints of a radio multicast transmission where feedback reports are costly or even impossible. Based on this, several mobile multicast standards have introduced FEC protection in the application layer (AL-FEC) of their multicast services boosted by powerful AL-FEC codes, i.e. Raptor codes family [1], that have recently emerged.

However, FEC protection comes with its own cost since controlling the introduced redundancy is not a trivial issue. The multicast sender should decide on the amount of the redundancy that should be introduced on the transmission so as to ensure that the recipients will be able to reconstruct the transmitted object recovering arbitrary data losses. At the same time the redundant information should be adapted to the current reception conditions in order to avoid resources wastage. It is obvious, especially on mobile environments, that the design of a feedback-report mechanism aiming to control the FEC encoding parameters is complicated due to the individual constraints of multicast environments or even impossible due to the variability of the radio propagation medium.

In some problems, where the application of deterministic solutions lacks of applicability, a randomized online algorithm [2] is the simplest available algorithm and sometimes the most efficient solution. Online algorithms are used to confront problems where the input of the algorithm is not available in advance. Subsequently, online algorithms have to generate output without knowledge of the entire input since input information arrives in the future and is not accessible at present. The effectiveness of online algorithms is evaluated using competitive analysis. The main concept of competitiveness is to compare the output generated by an online algorithm to the output produced by an optimal offline algorithm. The competitive ratio of a randomized online algorithm A is defined with respect to an adversary. In general, the adversary generates a sequence σ and the online algorithm A has to serve σ. When constructing the sequence σ, the adversary always knows the description of the online algorithm A. The optimal offline algorithm knows the entire request sequence in advance and can serve it with minimum cost. Formally, given a sequence σ, $A(\sigma)$ denotes the cost of the online algorithm A and $OPT(\sigma)$ denotes the cost of the optimal offline algorithm. An online algorithm A is called c-competitive if there exists a constant α such that $A(\sigma) - c \cdot OPT(\sigma) \leq \alpha$ [3].

Online algorithms are utilized in many research fields of mobile networks as the work presented in [4], where the frequency assignment problem is examined through distributed online algorithms. The work presented in [5] proposes a data selection policy where, the decision of transmitting source data, retransmitting a packet or transmitting a redundant codeword is investigated through competitive analysis. The work presented in [6] introduces a competitive online algorithm in terms of energy efficiency and delay in scheduling problems over wireless multicast environments. By reducing the energy-efficient transmission scheduling problem to a convex optimization problem the authors design a variety of online algorithms aiming to minimize the energy required to transmit packets in a wireless environment. Furthermore, the authors of [7] present a set of randomized online algorithms resolving the maximum independent set problem in disk graphs which can model resource allocation problems in mobile networks.

In this work we utilize online algorithms to achieve efficient deployment of AL-FEC protection over mobile multicast en-

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vironments. Our aim is to provide an AL-FEC deployment policy in order to face the main issue of argument in AL-FEC application, i.e. the efficient selection of the introduced transmission overhead. It is obvious that an arbitrary large amount of fixed overhead may lead to network resources wastage while a small amount of overhead may have no effect on the transmission robustness. To this direction, we introduce a novel AL-FEC deployment strategy based on randomized online algorithms and we utilize competitive analysis to demonstrate that the proposed strategy can operate surprisingly well under different loss patterns and network conditions without the need of costly feedback-report mechanisms.

The rest of this paper is organized as follows: In Section II we introduce the competitive framework describing the network model and the assumptions we utilize in our analysis. In Section III we present the proposed strategy on the AL-FEC protection deployment over mobile multicast transmission environments and in Section IV we analyze the performance of the proposed scheme. Finally, in Section V we conclude with a discussion on the advantages of the presented online scheme and we propose some possible future steps that could extend the presented work.

II. Competitive Framework

In this section we present the network model and the assumptions we utilize to introduce our proposed scheme on the AL-FEC application over mobile multicast environments.

The transmission environment we introduce refers to a typical mobile multicast environment. The same data are transmitted to a fraction of users participating in the multicast delivery through a shared unreliable radio channel. The transmitted data, considered to be discrete objects as in a download delivery session, are encapsulated in a UDP/IP multicast flow, with a multicast source injecting packets into the network.

Regarding the aim of this work, i.e. an efficient AL-FEC protection mechanism policy, we consider the application of the newly introduced RaptorQ FEC scheme [8]. The multicast sender introduces redundant information within the source data in order to enable multicast receivers to overcome independent packet losses and successfully reconstruct the source data. For the AL-FEC encoding, the transmitted object is partitioned in one or several source blocks. Each FEC source block consists of *k* source symbols with the assumption of one FEC symbol per packet of fixed length and with *k*, denoted as source block length (sbl), depending on the selection of the encoding parameters. Thereafter, a certain amount of redundant symbols, also called repair symbols, are generated according to the desired amount of protection introduced by the multicast source. For this purpose, RaptorQ encoding is used for each FEC source block. A unique ID is assigned to each resulting encoding symbol, which can be a source or a repair symbol, in order to identify the type of the symbol according to the assigned value. At the receiver side, a multicast client is able to determine, for each FEC source block, which source symbols should have been received but have not as well as the number of encoding symbols it has received.

RaptorQ FEC is the newest member of Raptor codes family providing powerful capabilities on the AL-FEC protection application. RaptorQ code is a fountain code, meaning that as many encoding symbols as desired can be generated by the encoder on-the-fly from the source symbols of a source block of data. The encoding process is systematic since n encoding symbols are produced from $k<n$ source symbols, so as the original source symbols are within the stream of the transmitted symbols. RaptorQ can encode up to 56403 source symbols into a source block and can generate up to 16777216 encoding symbols from the source symbols providing the ability to deliver files up to 3.4 GB as a single source block. The encoding process of such a FEC code provides the ability to the decoder to recover the whole source block from any set of encoding symbols only slightly more in number than the source symbols. More precisely, the performance of an AL-FEC code can be described by the decoding failure probability of the code, denoting the probability the RaptorQ decoder to fail on successfully reconstructing the protected data as a function of the source block size and the number of received symbols. The decoding failure probability of RaptorQ code can be modeled by (1) [9]:

$$p_{f_{RQ}}(n,k) = \begin{cases} 1, & \text{if } n < k \\ 0.01 \times 0.01^{n-k}, & \text{if } n \geq k \end{cases} \tag{1}$$

In (1), $p_{f_{RQ}}(n,k)$ denotes the probability of a failed decode of a RaptorQ protected block with k source symbols if n encoding symbols have been received.

It is clear that the performance of such an AL-FEC scheme strongly depends on the packet loss patterns that it has to cope with and on the amount of the redundancy introduced by the multicast source. In this work, we assume the transmission of a packet sequence with independent packet loss masks applied to each multicast receiver according to an examined packet loss rate. In each packet sequence, each packet is denoted by the triplet $\{uid, sbn, (r_i, l)\}$ where:

- uid: is a unique ID identifying each AL-FEC resulting packet
- sbn: is the number of the FEC source block the examined packet is organized to
- (r_i, l): defines if the examined packet was not received by the receiver i with the boolean l set to 0 if packet was not received

The packet loss pattern applied to the sequence of transmitted packets is denoted by p, which is the average network packet loss rate taking values in the range $[0, 1]$. At each multicast receiver, a packet loss mask is applied independently based on the value of p. The packet erasures are randomly distributed at the receivers as illustrated in Fig. 1, where an instance of the successful or not reception of 10 transmitted packets at 5 multicast receivers is presented. Value p is set to 0.2 inline with the assumptions of the previously described network model. Moreover, the packet loss mask is randomly distributed at the whole fraction of the transmitted object.

User 1	(1,0)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,0)	(1,0)
User 2	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)	(2,1)
User 3	(3,1)	(3,0)	(3,1)	(3,1)	(3,0)	(3,0)	(3,0)	(3,1)	(3,1)	(3,0)
User 4	(4,1)	(4,1)	(4,1)	(4,1)	(4,1)	(4,1)	(4,0)	(4,1)	(4,1)	(4,1)
User 5	(5,1)	(5,1)	(5,1)	(5,0)	(5,1)	(5,1)	(5,1)	(5,1)	(5,1)	(5,1)

Fig. 1. An Instance of Packet Erasures

At each multicast receiver the AL-FEC decoding process is modeled according to the decoding failure probability of (1) in order to determine whether the examined AL-FEC source block has been successfully reconstructed or not. On the decoding process, we assume that a sufficient threshold for the failure probability of a recovered source block is 10^{-2} or less as proposed in [10].

III. Online Algorithms for AL-FEC Deployment

In this section we present the proposed AL-FEC deployment policy based on online algorithms under the previous described network model. The present analysis is conducted considering the transmission cost of an AL-FEC protection scheme based on the introduced redundancy in the transmission. Since the multicast transmitter should decide on the amount of redundant information introduced to the transmitted data through the FEC encoding process, the problem addressed in this work is a cost minimization problem described by the number of packets transmitted in total including the number of redundant FEC packets.

On the AL-FEC policy problem the multicast source has to introduce an amount of transmission overhead having no knowledge on the packet losses pattern of the network. The multicast source has to minimize the number of transmitted data and at the same time to enable as many mobile users as possible to recover the transmitted object with respect to the packet erasures of each individual user. To define the problem, we assume that an AL-FEC encoder takes as input a sequence of packets and has to organize it in FEC source blocks of k FEC symbols, producing a certain amount of r repair symbols per source block according to the selected transmission overhead.

A. Optimal AL-FEC Policy Algorithm

Considering the cost of the optimal offline algorithm, we assume that a multicast transmitter, with a priori knowledge of the packet losses pattern of the network, will introduce a certain number of redundant symbols. Since the AL-FEC policy problem is a cost minimization problem, the optimal policy from the transmission cost perspective on the selection of the AL-FEC overhead which a multicast source should introduce to the transmission is not the introduction of a huge amount of overhead aiming to enable the higher packet loss user to successfully recover the transmitted object. Subsequently, the optimal algorithm will introduce the minimum number of required redundant symbols so as to cope with the average value of packet loss aiming to satisfy as many users as possible.

The scheme that can ensure the optimal selection of the transmission overhead is described by a multicast source that selects the introduced redundancy to a value close to the average packet loss rate of the network as defined in [11] given the recovery properties of the utilized AL-FEC code. In the present analysis the multicast sender can exploit the exceptional recovery properties of RaptorQ code. RaptorQ provides a practically zero reception overhead since, as described in (1), can achieve the specified threshold of the decoding failure probability requiring to receive no more additional encoding symbols than the number of the transmitted source symbols. Subsequently, the optimal AL-FEC selection policy can introduce as many repair symbols as the average number of lost symbols in the multicast users. Based on this, the number of repair symbols r the optimal offline algorithm will introduce in each source block of size k symbols is calculated as follows: $r = (k + r) \cdot p$.

Consequently, the cost of the optimal AL-FEC policy algorithm, defined as $OPT = k + r$, is described by (2):

$$OPT = k \cdot \left(1 + \frac{p}{1-p}\right) \tag{2}$$

B. Randomized AL-FEC Online Algorithm

In this paragraph we present a randomized online algorithm for the selection policy of the introduced AL-FEC redundancy on a multicast transmission. The proposed algorithm processes a sequence of packets according to the selected sbl, selecting equiprobably a value i, denoting the introduced transmission overhead, in the range $[0.05, 0.5]$ with a step of 0.01 when a source block is formed. Subsequently, the introduced transmission overhead is computed according to the random choice of i as described in Algorithm 1. The proposed online algorithm processes each packet and distributes it in the appropriate AL-FEC source block according to the selected sbl. At the last symbol of each source block the algorithm makes a random choice of the amount of redundant packets the AL-FEC encoder will produce for this particular block. Consequently, Algorithm 1 applies a random spread of the introduced overhead at all of the blocks that the transmitted object is divided into.

The proposed randomized online algorithm requires in fact just the input of the uid of the current packet and the selected length of each AL-FEC source block in order to distribute the packets in source blocks and to determine the amount of protection will be introduced in each formed block based on the random process described above.

The cost of the randomized online Algorithm 1 is expressed by (3), since the number of transmitted packets is $k + k \cdot i$ per source block:

$$ALG_1 = k \cdot \big(1 + E(I)\big) \tag{3}$$

Since the examined AL-FEC policy problem is a cost

Algorithm 1 A Randomized AL-FEC Policy Online Algorithm

procedure (pkt, sbl)
  $sbn \leftarrow \lfloor pkt.uid/sbl \rfloor$
  if $pkt.uid \bmod sbl \neq 0$ **then**
    $pkt.sbn \leftarrow sbn$
  else
    $pkt.sbn \leftarrow sbn$
    select equiprobably a value i from the set $\{0.05 : 0.01 : 0.5\}$
    transmission overhead $\leftarrow \lceil sbl * i \rceil$
  end if
end procedure

minimization problem, the competitive ratio c of the presented randomized online algorithm can be defined as the minimum value of c for which it applies:

$$E\big[ALG_1(\sigma)\big] - c \cdot OPT(\sigma) \leq \alpha \tag{4}$$

Therefore, since the definition of the AL-FEC policy problem allows to set the value of the quantity α equal to 0, the competitive ratio c of the randomized online Algorithm 1 can be calculated from (4) as:

$$c = \max \frac{ALG_1}{OPT}$$

Subsequently, given that the expected value of the random variable I, $E(I)$ is 0.275, the competitive ratio of Algorithm 1 is:

$$c = 1.275 \cdot (1 - p)$$

It is obvious that the performance of the proposed online algorithm depends on the value of the packet loss rate p introduced by an adversary. However, our objective is not to analyze the worst-case competitive ratio, but to investigate the performance of the randomized algorithm on typical packet loss scenarios.

At this point, we have to highlight that the optimal algorithm has prior knowledge of the packet loss conditions in contrast to the online algorithm which does not know or can not predict the outcome of the transmitted packets. This is why a deterministic algorithm can not be applied and we investigate the performance of an a priori arbitrary choice of the introduced redundancy at a fair range of values.

IV. Performance Analysis

In this section we provide simulation results evaluating the achieved performance of the proposed randomized online AL-FEC policy algorithm compared to the performance of the optimal policy algorithm over several perspectives. For the conducted evaluation we utilize 100 multicast users with the average packet loss rate over all mobile users varying between 1% and 10%.

A. Recovered AL-FEC Source Blocks

This paragraph presents simulation results for the amount of successfully decoded AL-FEC source blocks over all the multicast users. Fig. 2 presents the total number of recovered source blocks as a function of the selected packet loss rate values. For this evaluation we transmit 4 AL-FEC source blocks each one of size 128 source symbols.

By observing the plotted curves of Fig. 2 we can notice that the performance of the randomized online algorithm depends on the packet loss conditions that it has to confront. For low values of packet loss rate the online algorithm can operate surprisingly well, while as packet loss rate increase even more users fail to recover the transmitted source blocks. The optimal algorithm achieves a much more stable performance, successfully recovering a sufficient number of source blocks across the whole range of the evaluated packet loss rate. Furthermore, we can observe that for low values of packet loss rate the randomized online algorithm outperforms the optimal algorithm. This fact is anticipated since, given that the AL-FEC policy problem is a cost minimization problem, the optimal offline algorithm exploits its a priori knowledge of the packet loss conditions and introduces a certain amount of redundancy according to the average value of packet loss. Subsequently, the optimal algorithm is able to achieve an almost constant number of satisfied multicast receivers by adapting the introduced redundancy to the current network conditions. The form of the randomized online algorithm can be justified by the range of the randomly selected values of the introduced redundancy. However, we can observe that the online algorithm can cope well enough with a wide range of packet losses. On the other hand, a fixed overhead policy could only operate successfully under a limited range of packet loss rates and could lead to huge waste of network resources.

To further clarify the advantages of the proposed online scheme against a fixed AL-FEC overhead policy, in Fig. 3 we provide simulation results for the average number of successfully decoded source blocks per multicast user, transmitting

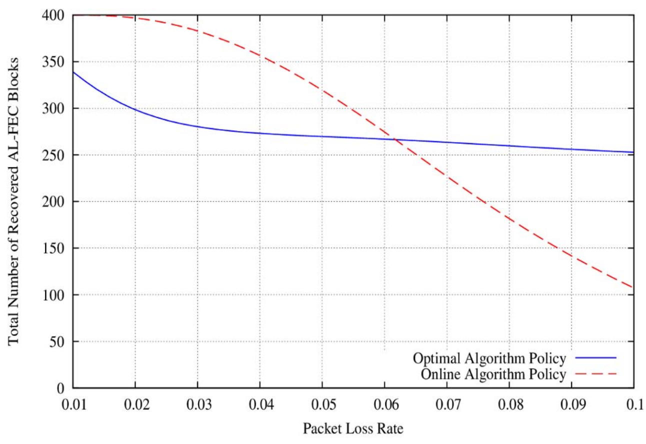

Fig. 2. Total Number of Recovered Source Blocks vs. Packet Loss Rate

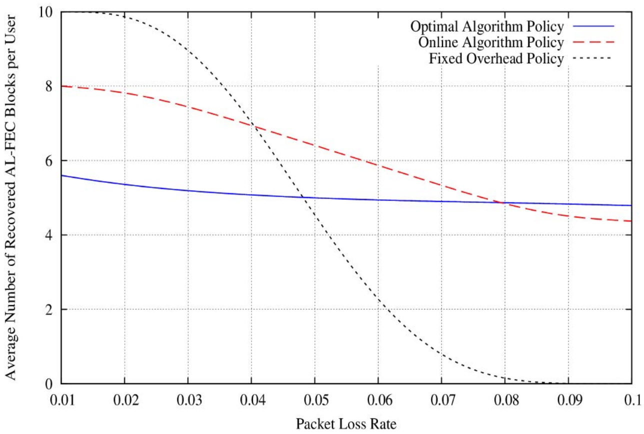

Fig. 3. Average Number of Recovered Source Blocks per User vs. Packet Loss Rate

an object of 10 source blocks each consisting of 128 source symbols. Fig. 3 compares the performance of the randomized online AL-FEC policy with that of a fixed overhead policy with the introduced AL-FEC overhead fixed at 5%.

The form of the plotted curves in Fig. 3 immediately reveals the advantages offered by the random spread of the proposed online AL-FEC policy against the application of AL-FEC protection with fixed overhead. We can observe that the 5% fixed overhead policy can provide sufficient protection only in a small range of packet loss rate since for values greater than 6% it can achieve successful decoding of just 1 AL-FEC source block in average from the 10 transmitted.

This fact is not surprising since the fixed introduced overhead dictates that the AL-FEC protection can be efficient only for values of packet loss rate close to the fixed overhead. For packet losses lower than the introduced overhead the fixed policy may indeed achieve a robust transmission but with a huge waste on network resources while, at the same time it is obvious that the fixed overhead is not capable to confront higher values of packet loss rate. On the other hand, the proposed randomized AL-FEC policy can operate fairly well under a wider range of packet loss rate achieving protection for different values of losses and providing generally a "stable" protection efficiency.

B. Total Number of Transmitted AL-FEC Symbols

In this subsection we evaluate the total number of transmitted AL-FEC symbols, including the repair symbols, that each AL-FEC policy introduces to the multicast transmission. Fig. 4 presents the total number of produced AL-FEC symbols of the online and the optimal algorithm as a function of the packet loss rate. The transmitted object is formed, as in the previous subsection, by 4 source blocks of 128 source symbols each one.

The plotted curves of Fig. 4 directly reflect the overhead policy mode of the optimal offline algorithm and the proposed randomized online algorithm. In case of the optimal offline algorithm the number of transmitted symbols, including the amount of introduced redundancy, increases in proportion to the average packet loss rate of the network. This increase directly results from the described optimal policy since the offline algorithm exploits its knowledge of the packet loss conditions and adapts the introduced redundancy accordingly.

On the other hand, we observe that the random overhead online algorithm introduces in average a nearly constant amount of redundant AL-FEC symbols about to 650 AL-FEC symbols varying between 130 and 180 more transmitted symbols in contrast to the amount of symbols transmitted by the offline optimal algorithm. However, the fact that the randomized online algorithm transmits a close to constant number of redundant symbols does not imply that the online algorithm operates as a fixed overhead policy. By examining several instances of the online algorithm operation it is obvious that it can introduce a wide range of transmission overhead in the hope of a packet loss match. This is indeed the aim of the randomized algorithm since with no knowledge on the network's condition and with the sbl fixed, it attempts to achieve a sufficient performance with a random spread of the introduced overhead over different AL-FEC source blocks.

C. AL-FEC Decoding Failure Probability

In this part of simulation results we examine the impact of the sbl increase on the average AL-FEC decoding failure probability of the transmitted AL-FEC source blocks. For the conduction of the presented results we utilize 4 transmitted source blocks of length {128, 256, 512, 1024, 2048, 4096}, with the average packet loss rate fixed at 8%. Fig. 5 presents the performance in terms of decoding failure probability of the proposed online and the optimal algorithm by simulating the transmission of an object of fixed size which is segmented in source blocks according to the evaluated values of sbl.

By observing the performance results presented in Fig. 5 we can extract some very interesting remarks. It is immediately apparent that both algorithms achieve improved performance

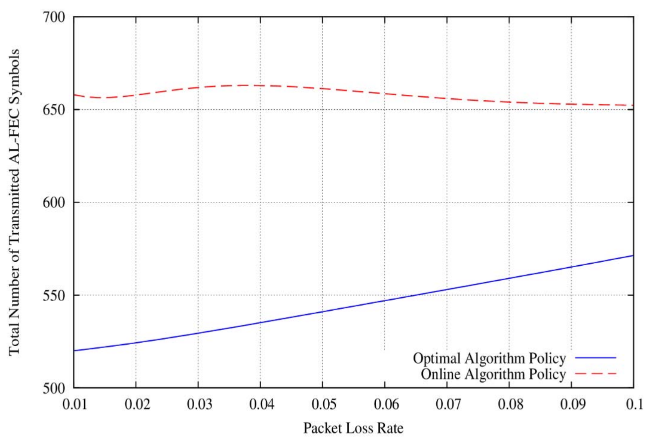

Fig. 4. Transmitted Symbols vs. Packet Loss Rate

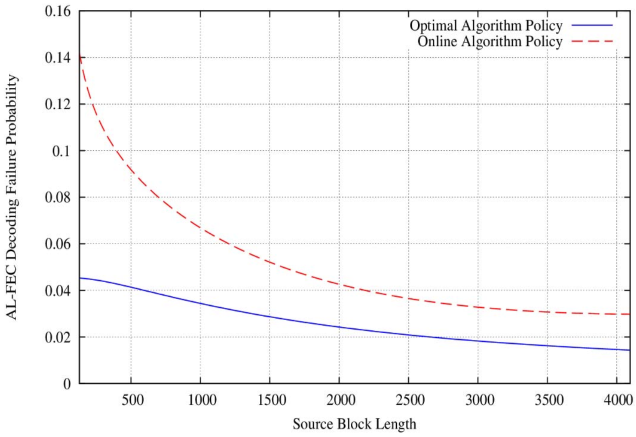

Fig. 5. Decoding Failure Probability vs. Source Block Length

by increasing the number of AL-FEC symbols protected together in a source block. However, we can remark that the increase of the selected sbl benefits more the performance of the randomized online algorithm in terms of achieved average AL-FEC failure probability than the optimal offline policy. More precisely, we can observe that increasing the sbl from 128 to 2048 symbols results in significant reduction of the decoding failure probability of the online algorithm from 0.14 to about 0.03, thus achieving a performance close enough to that of the optimal policy algorithm. The behavior of the online algorithm can be justified by the applied random spread of the introduced overhead on each AL-FEC source block, which benefits from the segmentation of the transmitted object into fewer and larger in size source blocks when sbl increases.

V. Conclusions & Future Work

In this work we have presented an online algorithm on the AL-FEC policy problem. We have examined a feedback-free scheme for the deployment of AL-FEC protection over mobile multicast environments utilizing RaptorQ FEC code. We have proposed a randomized online AL-FEC policy scheme aiming to effectively address the lack of knowledge of the packet loss conditions that the AL-FEC protection scheme has to confront, adopting a random selection policy on the introduced transmission overhead. The presented evaluation has been based on competitive analysis, examining the performance of the proposed online AL-FEC policy algorithm in comparison to an offline optimal algorithm with prior knowledge of packet loss patterns as a cost minimization problem.

We have introduced a realistic mobile multicast network model under which we have presented a detailed analysis on the competitiveness of the randomized online algorithm. We have demonstrated that the competitive ratio of the proposed online algorithm depends on the various packet loss conditions in a multicast environments. Thereafter, we have presented simulation results of the proposed online algorithm against the optimal policy algorithm examining several performance perspectives. Furthermore, we have compared the performance of the proposed randomized policy with that of a fixed overhead AL-FEC application. From the presented simulation results, we are able to verify the efficiency of the proposed online scheme and its superiority against a fixed AL-FEC overhead policy. We have demonstrated that the arbitrary random selection of the introduced AL-FEC transmission overhead can operate well enough in a reasonable range of packet loss rate, offering the possibility of the overhead spread and reducing the introduced redundancy through the exploitation of different AL-FEC encoding parameters.

Some possible future steps that can follow and extend this work are the design of a more sophisticated online scheme which, through prior knowledge of the network conditions, could choose the AL-FEC encoding parameters based on a prediction scheme. Furthermore, it is our belief that an online crosslayer scheme adapting the introduced redundancy on the application layer considering the amount of protection on lower layers could be beneficial for the robustness of a multicast transmission. Finally, we could introduce an online algorithm aiming to an efficient AL-FEC deployment over multicast streaming delivery taking into account the individual constraints of a streaming transmission.

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